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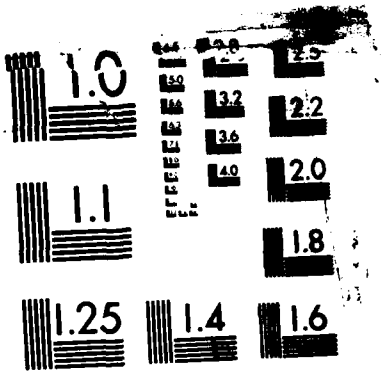


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# DAVID W. TAYLOR NAVAL SHIP RESEARCH AND DEVELOPMENT CENTER

Bethesda, Maryland 20084-5000


## STEADY SHIP WAVES AT LOW FROUDE NUMBERS (PART ONE)

by

Francis Noblesse

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SHIP PERFORMANCE DEPARTMENT
RESEARCH AND DEVELOPMENT REPORT

April 1986 DTNSRDC-86/024


STEADY SHIP WAVES AT LOW FROUDE NUMBERS (PART ONE)

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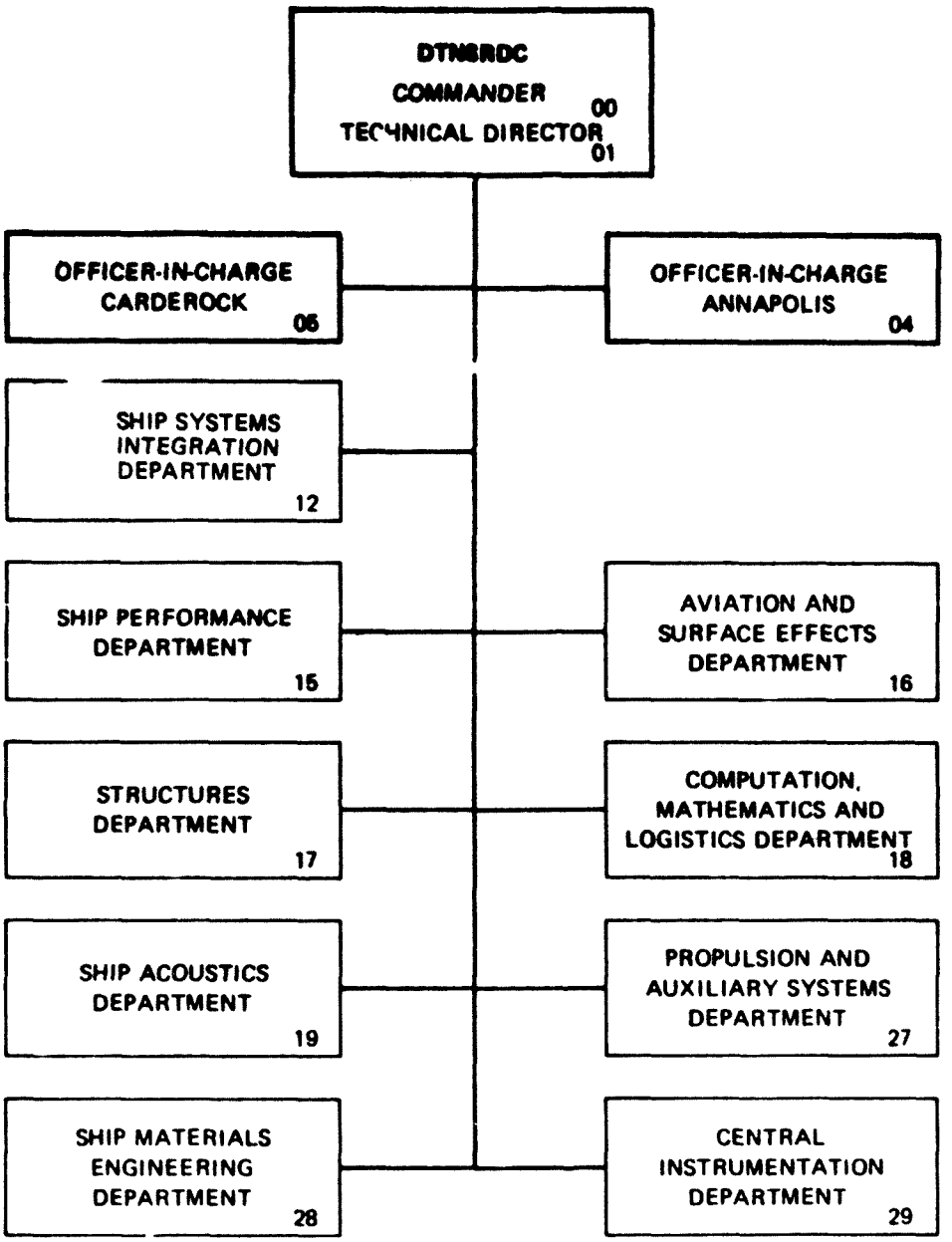


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A simple practical numerical method for estimating the wave resistance and the wave pattern of a ship operating at low Froude number is presented. The method is based upon approximating the two-fold integral over the ship-hull surface involved in the usual basic expression for the far-field wave-amplitude function by a one-fold integral around the ship mean waterline.

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## ABSTRACT

A simple practical numerical method for estimating the wave resistance and the wave pattern of a ship operating at low Froude number is presented. The method is based upon approximating the two-fold integral over the ship-hull surface involved in the usual basic expression for the far-field wave-amplitude function by a one-fold integral around the ship mean waterline.

## ADMINISTRATIVE INFORMATION

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## INTRODUCTION

A numerical method for evaluating the near- and far-field wave potential and the wave resistance of a ship advancing with constant speed in calm water within the theoretical framework of the Neumann-Kelvin theory developed in Noblesse [1] was recently presented in Barnell and Noblesse [2]. The numerical results obtained in the latter study show that a very large number of panels must be used for approximating the ship-hull surface for small values of the Froude number. The numerical method presented in [2] therefore becomes less practical at small Froude numbers, as is indeed noted in the conclusion to [2].

A complementary numerical method useful for small values of the Froude number is presented in this study. The method is based upon approximating the two-fold integral over the ship-hull surface involved in the usual basic expression for the wave-amplitude function by a one-fold integral around the ship mean waterline.

The low-Froude-number numerical method presented in this study can be further simplified and rendered more efficient by using an analytical approximation for the one-fold integral around the ship waterline obtained here. This analytical approximation, valid for sufficiently-small values of the Froude number, is presented in Part 2 of this study. Numerical applications will be presented in Part 3.

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*A complete listing of references is given on page 17.

## BASIC EXPRESSIONS FOR THE FAR-FIELD WAVE-AMPLITUDE FUNCTION

This study considers the steady potential flow due to a ship advancing with constant speed in calm water of infinite depth and lateral extent. Nondimensional coordinates $\vec{x} = \vec{X}/L$ and flow variables are used, with the length L and the speed U of the ship and the density $\varrho$ of water selected for reference. The mean free surface is taken as the plane $z = 0$, with the z axis pointing upwards, and the x axis is chosen in the ship centerplane and pointed towards the bow.

Equations (42) and (32) in [1] yield the following expressions for the nondimensional wave resistance $r \equiv R/\varrho U^2 L^2$ and the nondimensional velocity potential $\phi \equiv \Phi/UL$ far behind the ship:

$$\pi r = \int_0^\infty || K(t) ||^2 p \, dt, \tag{1}$$

$$\pi\phi(\vec{\xi}) \sim \int_0^\infty \mathrm{Im}[E_+(t;\vec{\xi}) + E_-(t;\vec{\xi})] \, K(t) \, dt, \tag{2}$$

where $E_\pm(t;\vec{\xi})$ represents the exponential function

$$E_\pm(t;\vec{\xi}) = \exp[\nu^2 p\{p\zeta + i(\xi \pm t\eta)\}]; \tag{2a}$$

Furthermore, p is defined as

$$p = (1 + t^2)^{1/2}, \tag{3}$$

$\nu$ is the inverse of the Froude number, that is

$$\nu = 1/F, \tag{4}$$

and K(t) represents the far-field wave-amplitude function defined by equation (36) in [1].

The function K(t) may be expressed in the form

$$K(t) = K_+(t) + K_-(t), \tag{5}$$

where the functions $K_\pm(t)$ are given by

$$K_\pm(t) = \int_c E_\pm(n_x^2 + t_x\phi_t - n_z t_y \phi_d + i\nu^2 p\phi) \, t_y \, dl + \nu^2 \int_h \exp(\nu^2 p^2 z) \, E_\pm(n_x + \nu^2 p^2 \phi n_\pm) \, da. \tag{6}$$

In this equation, $E_{\pm}$ represents the exponential function

$$E_{\pm} = \exp\,[-i\nu^2 p(x \pm ty)]; \tag{7}$$

Furthermore, c and h represent the positive halves of the mean waterline and of the mean wetted-hull surface, respectively. The unit vector tangent to c and pointing towards the bow is denoted by $\vec{t}(t_x,t_y,0)$, and $\vec{n}(n_x,n_y,n_z)$ is the unit vector normal to h and pointing into the water, as is indicated in figure 1. The term $n_{\pm}$ is defined as

$$n_{\pm} = -n_z + i(n_x \pm t\, n_y)/p. \tag{8}$$

Also, dl and da represent the differential elements of arc length and of area of c and h, respectively. Finally, $\phi \equiv \phi(\vec{x})$ represents the disturbance potential at the integration point $\vec{x}$ on c or h, $\phi_t$ represents the derivative of $\phi$ in the direction of the tangent vector $\vec{t}$ to c, and $\phi_d$ is the derivative of $\phi$ in the direction of the vector $\vec{n} \times \vec{t}$, which is tangent to h and pointing downwards as is shown in figure 1.

For large values of $\nu$, or more generally of $\nu p$, the exponential function $\exp(\nu^2 p^2 z)$ in the hull-surface integral in equation (6) vanishes rapidly for negative values of z. Therefore, only the upper part of h yields a significant contribution and the hull-surface integral can be approximated by a line integral along c in the low-Froude-number limit. This low-Froude-number asymptotic approximation of the far-field wave-amplitude function in terms of a waterline integral is obtained in the following section.

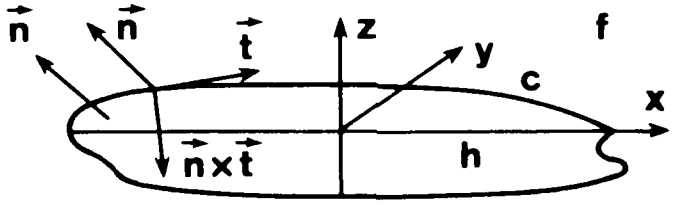


Figure 1 - Definition Sketch for a Single-Hull Ship with Port and Starboard Symmetry

## LOW-FROUDE-NUMBER APPROXIMATION TO THE FAR-FIELD WAVE-AMPLITUDE FUNCTION IN TERMS OF A WATERLINE INTEGRAL

The mean waterline c can be represented by the parametric equations

$$x = x_0(\lambda) \text{ and } y = y_0(\lambda), \tag{9a,b}$$

where the parameter $\lambda$ varies between its bow and stern values, that is $\lambda_B \leq \lambda \leq \lambda_S$. In the vicinity of c, the hull surface may then be represented by the parametric equations

$$x = x_0(\lambda) + x_1(\lambda)z + x_2(\lambda)z^2 + \ldots \tag{10a}$$

$$y = y_0(\lambda) + y_1(\lambda)z + y_2(\lambda)z^2 + \ldots \tag{10b}$$

where $\lambda_B \leq \lambda \leq \lambda_S$ and $z \leq 0$.

Equations (7) and (10a,b) yield

$$\exp(\nu^2 p^2 z)\, E_{\pm} = E_0{}^{\pm} \exp[\nu^2 p^2 (1 - i\gamma_1{}^{\pm})z] \cdot \exp[-i\nu^2 p^2 (\gamma_2{}^{\pm} z^2 + \gamma_3{}^{\pm} z^3 + \ldots)], \tag{11}$$

where $E_0{}^{\pm}$ represents the exponential function

$$E_0{}^{\pm} = \exp(-i\nu^2 p^2 \gamma_0{}^{\pm}), \tag{12}$$

and $\gamma_n{}^{\pm}$, $n \geq 0$, is defined as

$$\gamma_n{}^{\pm} = (x_n \pm t\, y_n)/p. \tag{13}$$

The Taylor series of the exponential function $\exp[-i\nu^2 p^2 (\gamma_2{}^{\pm} z^2 + \gamma_3{}^{\pm} z^3 + \ldots)]$ is

$$\exp[-i\nu^2 p^2 (\gamma_2{}^{\pm} z^2 + \gamma_3{}^{\pm} z^3 + \ldots)] = 1 - i\nu^2 p^2 z^2 \gamma_2{}^{\pm} - \nu^2 p^2 z^3 [i\gamma_3{}^{\pm} + \nu^2 p^2 z (\gamma_2{}^{\pm})^2/2] + \ldots \tag{14}$$

The parametric representation (10a,b) of the hull surface yields

$$\vec{n}\, da = \vec{m}\, d\lambda\, dz, \tag{15}$$

where the normal vector $\vec{m}$ to h is given by

$$\vec{m} = (\partial \vec{x}/\partial \lambda) \times (\partial \vec{x}/\partial z). \tag{15a}$$

Equations (10a,b) then yield

$$\vec{m} = \vec{m}_0 + \vec{m}_1 z + \vec{m}_2 z^2 + \ldots, \tag{16}$$

where the vectors $\vec{m}_n$ have components

$$\vec{m}_n = (m_n{}^x, m_n{}^y, m_n{}^z)$$

given by

$$m_n{}^x = y_n{}', \quad m_n{}^y = -x_n{}', \tag{16a,b}$$

$$m_0{}^z = x_0{}' y_1 - y_0{}' x_1, \tag{16c}$$

$$m_1{}^z = x_1{}' y_1 - y_1{}' x_1 + 2(x_0{}' y_2 - y_0{}' x_2), \tag{16d}$$

$$m_2{}^z = x_2{}' y_1 - y_2{}' x_1 + 2(x_1{}' y_2 - y_1{}' x_2) + 3(x_0{}' y_3 - y_0{}' x_3). \tag{16e}$$

In equations (16a-e), and hereafter in this study, the superscript $'$ denotes differentiation with respect to $\lambda$. Thus, we have $x_n{}' = dx_n(\lambda)/d\lambda$.

The potential $\phi(\lambda,z)$ on h in the vicinity of c may be expressed in the form

$$\phi = \phi_0(\lambda) + \phi_1(\lambda)z + \phi_2(\lambda)z^2 + \ldots . \tag{17}$$

Equation (15) yields

$$(n_x + \nu^2 p^2 \phi n_{\pm})\, da = (m_x + \nu^2 p^2 \phi m_{\pm})\, d\lambda\, dz, \tag{18}$$

where we have

$$m_{\pm} = -m_z + i(m_x \pm t\, m_y)/p \tag{19}$$

in accordance with equation (8). By using equations (16) and (17) we may obtain

$$\begin{aligned} m_x + \nu^2 p^2 \phi m_{\pm} &= \nu^2 p^2 \phi_0 m_0{}^{\pm} \\ &+ [m_0{}^x + \nu^2 p^2 z\, (\phi_0 m_1{}^{\pm} + \phi_1 m_0{}^{\pm})] \\ &+ z[m_1{}^x + \nu^2 p^2 z(\phi_0 m_2{}^{\pm} + \phi_1 m_1{}^{\pm} + \phi_2 m_0{}^{\pm})] + \ldots , \end{aligned} \tag{20}$$

where we have

$$m_n{}^{\pm} = -m_n{}^z + i(m_n{}^x \pm t\, m_n{}^y)/p. \tag{21}$$

Equations (20) and (14) then yield

$$\begin{aligned} &(m_x + \nu^2 p^2 \phi m_{\pm}) \exp[-i\nu^2 p^2(\gamma_2{}^{\pm} z^2 + \gamma_3{}^{\pm} z^3 + \ldots)] \\ &= \nu^2 p^2 \phi_0 m_0{}^{\pm} + F_1{}^{\pm} + z\, F_2{}^{\pm} + \ldots , \end{aligned} \tag{22}$$

where we have

$$\begin{aligned} F_1{}^{\pm} &= m_0{}^x + \nu^2 p^2 z(\phi_0 m_1{}^{\pm} + \phi_1\, m_0{}^{\pm}) \\ &\quad - i\nu^4 p^4 z^2 \phi_0 m_0{}^{\pm} \gamma_2{}^{\pm}, \end{aligned} \tag{22a}$$

$$F_2^{\pm} = m_1^x + \nu^2 p^2 z(\phi_0 m_2^{\pm} + \phi_1 m_1^{\pm} + \phi_2 m_0^{\pm} - i\, m_0^x \gamma_2^{\pm})$$
$$- i\nu^4 p^4 z^2[\phi_0(m_1^{\pm}\gamma_2^{\pm} + m_0^{\pm}\gamma_3^{\pm}) + \phi_1 m_0^{\pm}\gamma_2^{\pm}]$$
$$- \nu^6 p^6 z^3 \phi_0 m_0^{\pm}(\gamma_2^{\pm})^2/2. \tag{22b}$$

Equations (11), (18) and (22) yield

$$\exp(\nu^2 p^2 z)\, E_{\pm}(n_x + \nu^2 p^2 \phi n_{\pm})\, da =$$
$$E_0^{\pm}(\nu^2 p^2 \phi_0 m_0^{\pm} + F_1^{\pm} + zF_2^{\pm} + \ldots)\exp[\nu^2 p^2(1 - i\gamma_1^{\pm})z]\, d\lambda\, dz.$$

We then have

$$\int_h \exp(\nu^2 p^2 z)\, E_{\pm}(n_x + \nu^2 p^2 \phi n_{\pm})\, da = \int_{\lambda_B}^{\lambda_S} E_0^{\pm} I_{\pm}\, d\lambda, \tag{23}$$

where $I_{\pm}$ represents the integral

$$I_{\pm} = \int^0 (\nu^2 p^2 \phi_0 m_0^{\pm} + F_1^{\pm} + zF_2^{\pm} + \ldots)\exp[\nu^2 p^2(1 - i\gamma_1^{\pm})z]\, dz. \tag{24}$$

The lower limit of integration in the integral (24) may be taken equal to $-\infty$. We have

$$\int_{-\infty}^0 z^n \exp[\nu^2 p^2(1 - i\gamma_1^{\pm})z]\, dz = (-1)^n n!\, F^{2n+2} q^{2n+2} (u_{\pm})^{n+1}, \tag{25}$$

where q and $u_{\pm}$ are defined as

$$q = 1/p = (1 + t^2)^{-1/2}, \tag{26}$$

$$u_{\pm} = 1/(1 - i\gamma_1^{\pm}) = [1 - i(x_1 \pm t y_1)/p]^{-1}. \tag{27}$$

Equations (3) and (13) were used in equations (26) and (27), respectively. By using equations (22a,b) and (25) in equation (24), we may obtain

$$I_{\pm} = u_{\pm}\phi_0 m_0^{\pm} + F^2 q^2 u_{\pm} G_1^{\pm} + F^4 q^4 (u_{\pm})^2 G_2^{\pm} + \ldots, \tag{28}$$

where $G_1^{\pm}$ and $G_2^{\pm}$ are defined as

$$G_1^{\pm} = y_0' - u_{\pm}(\phi_0 m_1^{\pm} + \phi_1 m_0^{\pm}) - 2i(u_{\pm})^2 \phi_0 m_0^{\pm}\gamma_2^{\pm}, \tag{28a}$$

$$G_2^{\pm} = -y_1' + 2u_{\pm}(\phi_0 m_2^{\pm} + \phi_1 m_1^{\pm} + \phi_2 m_0^{\pm} - iy_0'\gamma_2^{\pm})$$
$$+ 6i(u_{\pm})^2[\phi_0(m_1^{\pm}\gamma_2^{\pm} + m_0^{\pm}\gamma_3^{\pm}) + \phi_1 m_0^{\pm}\gamma_2^{\pm}]$$
$$- 12(u_{\pm})^3 \phi_0 m_0^{\pm}(\gamma_2^{\pm})^2. \tag{28b}$$

Equations (17) and (10a,b) yield

$$\phi(x_0 + x_1 z + \ldots, y_0 + y_1 z + \ldots, z) =$$

$$\phi(x_0, y_0, 0) + \phi_1 z + \ldots .$$

By expanding the potential in a Taylor series about the point $(x_0, y_0, 0)$, we may then obtain

$$\phi_1 = x_1 \phi_x + y_1 \phi_y + \phi_z, \tag{29}$$

where $\phi_x$, $\phi_y$, $\phi_z$ represent the derivatives of $\phi$ with respect to x, y, z. We have

$$\phi_x = \nabla\phi \cdot \vec{i} = (\phi_n \vec{n} + \phi_t \vec{t} + \phi_d \vec{n} \times \vec{t}) \cdot \vec{i},$$

where $\vec{i}$ represents the unit vector along the x axis, and $\phi_n$ is the derivative of $\phi$ in the outward normal direction $\vec{n}$ to h. We then have

$$\phi_x = n_x \phi_n + t_x \phi_t - n_z t_y \phi_d. \tag{30a}$$

We may similarly obtain

$$\phi_y = n_y \phi_n + t_y \phi_t + n_z t_x \phi_d, \tag{30b}$$

$$\phi_z = n_z \phi_n + (n_x t_y - n_y t_x) \phi_d. \tag{30c}$$

Let D be defined as

$$D = t_x n_y - t_y n_x + (x_1 t_y - y_1 t_x) n_z. \tag{31}$$

Equations (29), (30a,b,c) and (31) then yield

$$\phi_d D = (x_1 t_x + y_1 t_y) \phi_t - \phi_1,$$

where the relation $x_1 n_x + y_1 n_y + n_z = \vec{n} \cdot \partial \vec{x}(\lambda, z)/\partial z = 0$ was used. We then have

$$n_x^2 + t_x \phi_t - n_z t_y \phi_d + i\nu^2 p\phi = i\nu^2 p\phi + n_x^2 + \phi_1 n_z t_y / D$$

$$+ [t_x - (x_1 t_x + y_1 t_y) n_z t_y / D] \phi_t. \tag{32}$$

On the mean waterline we have $z = 0$; equations (15), (16) and (16a,b,c) then yield

$$n_x = y_0' u (1 + \varepsilon^2)^{1/2}, \tag{33a}$$

$$n_y = -x_0' u (1 + \varepsilon^2)^{1/2}, \tag{33b}$$

$$n_z = \varepsilon (1 + \varepsilon^2)^{1/2}, \tag{33c}$$

where u and $\varepsilon$ are defined as

$$u = [(x_0')^2 + (y_0')^2]^{1/2}, \tag{34}$$

$$\varepsilon = (x_0' x_1 - x_0' y_1)/u. \tag{35}$$

Furthermore, we have

$$\phi = \phi_0, \tag{36}$$

$$dl = u\, d\lambda, \tag{37}$$

$$t_x = -x_0'/u, \qquad t_y = -y_0'/u, \tag{38a,b}$$

$$\phi_t = -\phi_0'/u, \tag{39}$$

where $\phi_0' = d\phi_0(\lambda)/d\lambda$. Equations (32), (31), (33a,b,c), (34), (35), (36), (37), (38a,b) and (39) then yield

$$(n_x^2 + t_x\phi_t - n_z t_y \phi_d + i\nu^2 p\phi)t_y\, dl = -Iy_0'\, d\lambda, \tag{40}$$

where I is given by

$$I = i\nu^2 p\phi_0 + H_1/u^2(1+\varepsilon^2) \tag{41}$$

with $H_1$ defined as

$$H_1 = (y_0')^2 + \varepsilon y_0' u\phi_1 + [(1+\varepsilon^2)x_0' - \varepsilon y_0'(x_1 x_0' + y_1 y_0')/u]\phi_0'. \tag{42}$$

Equations (40), (7), (9a,b), (12) and (13) then yield

$$\int_C E_\pm (n_x^2 + t_x\phi_t - n_z t_y\phi_d + i\nu^2 p\phi)t_y\, dl = -\int_{\lambda_B}^{\lambda_S} E_0^\pm Iy_0'\, d\lambda. \tag{43}$$

Equations (6), (23) and (43) yield

$$K_\pm(t) = \int_{\lambda_B}^{\lambda_S} E_0^\pm A_\pm\, d\lambda, \tag{44}$$

where $A_\pm$ is given by

$$A_\pm = \nu^2 I_\pm - Iy_0'. \tag{45}$$

By using equations (28) and (41) in equation (45), we may obtain

$$A_\pm = \nu^2(u_\pm m_0^\pm - ipy_0')\phi_0 - y_0' H_1/u^2(1+\varepsilon^2) + q^2 u_\pm G_1^\pm + F^2 q^4 (u_\pm)^2 G_2^\pm + \ldots . \tag{46}$$

Equations (27), (21), (16a,b,c), (13) and (3) yield

$$u_{\pm}m_0^{\pm} - ipy_0' = \mp i(x_0' \pm ty_0')\,(t/p \mp iy_1)u_{\pm}.$$

Equations (12), (13) and (26) then yield

$$\nu^2 E_0^{\pm}(u_{\pm}m_0^{\pm} - ipy_0')\phi_0 = \pm q[E_0^{\pm}(tq \mp iy_1)u_{\pm}\phi_0]'$$

$$\mp qE_0^{\pm}[(tq \mp iy_1)u_{\pm}\phi_0]', \tag{47}$$

where the superscript ' denotes differentiation with respect to the parameter $\lambda$. By using equations (45), (46) and (47) in equation (44) we may obtain

$$K_{\pm}(t) = \pm q[E_0^{\pm}(tq \mp iy_1)u_{\pm}\phi_0]_{\lambda_B}^{\lambda_S}$$

$$+ q^2 \int_{\lambda_B}^{\lambda_S} E_0^{\pm} a_{\pm}\, d\lambda, \tag{48}$$

where $a_{\pm}$ is defined as

$$a_{\pm} = u_{\pm}a_1^{\pm} + F^2q^2(u_{\pm})^2 G_2^{\pm} + \ldots, \tag{49}$$

with $a_1^{\pm}$ given by

$$a_1^{\pm} = \mp p[(tq \mp iy_1)u_{\pm}\phi_0]'/u_{\pm}$$

$$- p^2 y_0' H_1/u^2(1+\varepsilon^2)u_{\pm} + G_1^{\pm}. \tag{50}$$

At the bow, we have $y = 0$ for $z \leq 0$; equation (10b) then shows that we have $y_n(\lambda) = 0$ for $\lambda = \lambda_B$ and $n \geq 0$. Equations (12), (13), (26) and (27) then yield

$$\pm qE_0^{\pm}(tq \mp iy_1)u_{\pm}\phi_0 = \pm tq^2[\exp(-i\nu^2 px_0)]\phi_0/(1 - iqx_1)$$

for $\lambda = \lambda_B$. Equation (5) then shows that the contribution of the first term on the right side of equation (48) to the function K(t) is null for $\lambda = \lambda_B$. The contribution of this term clearly is also null for $\lambda = \lambda_S$ in the usual case of a ship with a closed stern. For a potential-flow-model involving an open-stern ship-and-wake extending to downstream infinity we have $\phi_0 \to 0$ as $x \to -\infty$, and the first term on the right side of equation (48) therefore also vanishes for $\lambda = \lambda_S$. Equations (5) and (48) then yield

$$K(t) = q^2 \int_{\lambda_B}^{\lambda_S} (E_0^{+} a_{+} + E_0^{-} a_{-})\, d\lambda. \tag{51}$$

By using equations (42), (28a), (13), (27), (35), (21) and (16a-d) into equation (50) we may obtain, after some algebraic manipulations

$$a_1^{\pm} = y_0' A_{\pm}/(1+\varepsilon^2) + 2q(x_0' \pm t y_0')(u_{\pm})^2 B_{\pm}\phi_0 + C_{\pm}\phi_0' + uD_{\pm}\phi_1/(1+\varepsilon^2) + ip(y_1\phi_0' - y_0'\phi_1), \tag{52}$$

where $A_{\pm}$, $B_{\pm}$, $C_{\pm}$ and $D_{\pm}$ are defined as

$$A_{\pm} = [(1+py_0'/u)(1-py_0'/u)+\varepsilon^2] + i(py_0'/u)[y_1(x_0' \pm t y_0')/u+\varepsilon], \tag{53a}$$

$$B_{\pm} = q(y_2 \mp t x_2) + i(y_1 x_2 - x_1 y_2), \tag{53b}$$

$$C_{\pm} = [(\mp t - p^2 x_0' y_0'/u^2) + (py_0'/u)^2 \varepsilon(x_1 x_0' + y_1 y_0')/u(1+\varepsilon^2)] + i[y_1(x_0' \pm t y_0')/u+\varepsilon][px_0'/u - (py_0'/u)\varepsilon(x_1 x_0' + y_1 y_0')/u(1+\varepsilon^2)], \tag{53c}$$

$$D_{\pm} = [(x_0' \pm t y_0')/u][(1+\varepsilon^2)(y_1 \pm iqt)u_{\pm} + i(py_0'/u)\varepsilon y_1] - (py_0'/u)\varepsilon(py_0'/u - i\varepsilon). \tag{53d}$$

In the particular case when $x_0' \pm t y_0' = 0$, equations (3), (34) and (35) yield $px_0'/u = -t$, $py_0'/u = \pm 1$, $(x_1 x_0' + y_1 y_0')/u = -q(tx_1 \mp y_1)$ and $\varepsilon = q(ty_1 \pm x_1)$. Equations (52) and (53a,c,d) then yield

$$a_1^{\pm} = \pm \varepsilon[iy_0' + (py_1 \mp it)\phi_0' \mp u\phi_1]/(1 \pm i\varepsilon) + ip(y_1\phi_0' - y_0'\phi_1) \quad \text{if } x_0' \pm t y_0' = 0. \tag{54}$$

Furthermore, if the hull surface intersects the free-surface plane orthogonally at the point $(x_0,y_0,0)$ of the waterline for which $x_0' \pm t y_0' = 0$, we have $n_z = 0$. Equation (33c) then yields $\varepsilon = 0$ and equation (54) becomes

$$a_1^{\pm} = ip(y_1\phi_0' - y_0'\phi_1) \quad \text{if } x_0' \pm t y_0' = 0 \text{ and } n_z = 0. \tag{55}$$

Equations (38a,b), (39), (29), the identity $\phi_t = t_x\phi_x + t_y\phi_y$, and equation (35) yield

$$(y_1\phi_0' - y_0'\phi_1)/u = -\varepsilon\phi_x + t_y\phi_z.$$

Equation (55) then becomes

$$a_1^{\pm} = iF^2 p y_0' \phi_{xx} \quad \text{if } x_0' \pm t y_0' = 0 \text{ and } n_z = 0. \tag{56}$$

Equations (49) and (56) then show that the amplitude function $a_{\pm}$ in equation (51) is of order $F^2$ at a point $(x_0,y_0,0)$ where the phase of the exponential function $E_0^{\pm} = \exp[-i\nu^2 p(x_0 \pm t y_0)]$ is stationary, that is where $x_0' \pm t y_0' = 0$, if the hull intersects the free-surface plane orthogonally at that point.

## SUMMARY OF RESULTS

The mean waterline is represented by the parametric equations

$$x = x_0(\lambda) \text{ and } y = y_0(\lambda), \tag{57}$$

where the parameter $\lambda$ varies between its bow and stern values, that is

$$\lambda_B \leq \lambda \leq \lambda_S. \tag{57a}$$

In the vicinity of the mean free surface, the hull surface is represented by the parametric equations

$$x = x_0(\lambda) + x_1(\lambda)z + x_2(\lambda)z^2 + \ldots, \tag{58a}$$

$$y = y_0(\lambda) + y_1(\lambda)z + y_2(\lambda)z^2 + \ldots, \tag{58b}$$

$$\text{where } \lambda_B \leq \lambda \leq \lambda_S \text{ and } z \leq 0. \tag{58c}$$

The velocity potential $\phi(\lambda,z)$ on the hull surface in the vicinity of the plane $z = 0$ likewise is expressed in the form

$$\phi = \phi_0(\lambda) + \phi_1(\lambda)z + \phi_2(\lambda)z^2 + \ldots . \tag{59}$$

Differentiation of the functions $x_n(\lambda)$, $y_n(\lambda)$, $\phi_n(\lambda)$ with respect to the parameter $\lambda$ is denoted by the superscript $'$; thus, we have $x_0' = dx_0(\lambda)/d\lambda$.

For sufficiently small values of the Froude number F, the far-field wave-amplitude function K(t) is given by the one-fold integral

$$K(t) = q^2 \int_{\lambda_B}^{\lambda_S} (E_0^{+} a_{+} + E_0^{-} a_{-})\, d\lambda, \tag{60}$$

where we have

$$q = 1/p = 1/(1+t^2)^{1/2}, \tag{61}$$

and $E_0^{\pm}$ and $a_{\pm}$ are the exponential function and the amplitude function defined below. The exponential function $E_0^{\pm}$ is given by

$$E_0^{\pm} = \exp[-i\nu^2 p(x_0 \pm t y_0)], \tag{62}$$

where $\nu$ is the inverse of the Froude number, that is

$$\nu = 1/F. \tag{63}$$

The amplitude function $a_{\pm}$ is given by

$$a_{\pm} = u_{\pm}a_1^{\pm} + F^2q^2(u_{\pm})^2a_2^{\pm} + \ldots, \tag{64}$$

where $u_{\pm}$ is defined as

$$u_{\pm} = 1/[1 - iq(x_1 \pm ty_1)], \tag{65}$$

and the functions $a_1^{\pm}$ and $a_2^{\pm}$ are now defined.

The first-order amplitude function $a_1^{\pm}$ is given by

$$\begin{aligned} a_1^{\pm} &= y_0'A_{\pm}/(1+\varepsilon^2) + 2q(x_0' \pm ty_0')(u_{\pm})^2B_{\pm}\phi_0 \\ &\quad + C_{\pm}\phi_0' + uD_{\pm}\phi_1/(1+\varepsilon^2) + ip(y_1\phi_0' - y_0'\phi_1), \end{aligned} \tag{66}$$

where we have

$$u = [(x_0')^2 + (y_0')^2]^{1/2}, \tag{67}$$

$$\varepsilon = (y_0'x_1 - x_0'y_1)/u, \tag{68}$$

and the coefficients $A_{\pm}$, $B_{\pm}$, $C_{\pm}$ and $D_{\pm}$ are defined as

$$\begin{aligned} A_{\pm} &= [(1 + py_0'/u)(1 - py_0'/u) + \varepsilon^2] \\ &\quad + i(py_0'/u)[y_1(x_0' \pm ty_0')/u + \varepsilon], \end{aligned} \tag{69a}$$

$$B_{\pm} = q(y_2 \mp tx_2) + i(y_1x_2 - x_1y_2), \tag{69b}$$

$$\begin{aligned} C_{\pm} &= [(\mp t - p^2x_0'y_0'/u^2) + (py_0'/u)^2\varepsilon(x_1x_0' + y_1y_0')/u(1+\varepsilon^2)] \\ &\quad + i[y_1(x_0' \pm ty_0')/u + \varepsilon][px_0'/u - (py_0'/u)\varepsilon(x_1x_0' + y_1y_0')/u(1+\varepsilon^2)], \end{aligned} \tag{69c}$$

$$\begin{aligned} D_{\pm} &= [(x_0' \pm ty_0')/u][(1+\varepsilon^2)(y_1 \pm iqt)u_{\pm} + i(py_0'/u)\varepsilon y_1] \\ &\quad - (py_0'/u)\varepsilon(py_0'/u - i\varepsilon). \end{aligned} \tag{69d}$$

The second-order amplitude function $a_2^{\pm}$ is given by

$$\begin{aligned} a_2^{\pm} &= -y_1' + 2u_{\pm}(\phi_0m_2^{\pm} + \phi_1m_1^{\pm} + \phi_2m_0^{\pm} - iy_0'\gamma_2^{\pm}) \\ &\quad + 6i(u_{\pm})^2[\phi_0(m_1^{\pm}\gamma_2^{\pm} + m_0^{\pm}\gamma_3^{\pm}) + \phi_1m_0^{\pm}\gamma_2^{\pm}] \\ &\quad - 12(u_{\pm})^3\phi_0m_0^{\pm}(\gamma_2^{\pm})^2, \end{aligned} \tag{70}$$

where we have

$$\gamma_n^{\pm} = q(x_n \pm ty_n), \tag{71}$$

$$m_n^{\pm} = \mu_n + iq(y_n' \mp tx_n'), \tag{72}$$

with

$$\mu_0 = \varepsilon u, \tag{72a}$$

$$\mu_1 = x_1 y_1' - y_1 x_1' + 2(x_2 y_0' - y_2 x_0'), \tag{72b}$$

$$\mu_2 = x_1 y_2' - y_1 x_2' + 2(x_2 y_1' - y_2 x_1') + 3(x_3 y_0' - y_3 x_0'). \tag{72c}$$

At a point where the phase of the exponential function $E_0^{\pm}$ is stationary, that is at a point $(x_0, y_0, 0)$ where we have $x_0' \pm t y_0' = 0$, the first-order amplitude function $a_1^{\pm}$ takes the form

$$a_1^{\pm} = \pm\varepsilon[iy_0' + (py_1 \pm it)\phi_0' \mp u\phi_1]/(1 \pm i\varepsilon) + ip(y_1\phi_0' - y_0'\phi_1)$$

$$\text{if } x_0' \pm t y_0' = 0. \tag{73}$$

Furthermore, if the hull surface intersects the plane $z = 0$ orthogonally at a point of stationary phase we have

$$a_1^{\pm} = iF^2 p y_0' \phi_{xx} \quad \text{if } x_0' \pm t y_0' = 0 \text{ and } n_z = 0, \tag{74}$$

and the amplitude function $a_{\pm}$ then is of order $F^2$.

The low-Froude-number approximation (60) for the far-field wave-amplitude function is well suited for numerical evaluation, as is shown in the next section. An analytical approximation to the integral (60) may also be obtained by taking advantage of the rapid oscillations of the exponential function $E_0^{\pm}$ in the low-Froude-number limit. This analytical approximation is presented in part 2 of this study.

For sufficiently small values of the Froude number, it may be acceptable for practical applications to approximate the velocity potential by its zero-Froude-number limit, in which the free surface becomes a rigid flat wall. This simple approximation for the potential, together with the low-Froude-number approximation for the far-field wave-amplitude function obtained in this study and the well-known integrals (1) and (2) defining the wave resistance and the far-field potential, provide a simple practical numerical method for estimating the wave resistance and the wave pattern of a ship operating at low Froude numbers.

## NUMERICAL EVALUATION OF THE LOW-FROUDE-NUMBER APPROXIMATION TO THE FAR-FIELD WAVE-AMPLITUDE FUNCTION

The functions $x_k(\lambda)$ and $y_k(\lambda)$, where $k = 0__3$, in equations (58a,b) can be determined numerically by considering the four waterlines $z = -k\delta$, where $k = 0__3$ and $\delta$ is sufficiently small in comparison with the (nondimensional) ship draft $d\ (= D/L)$; for instance, $\delta$ might be taken equal to $d/10$. Each of these four waterlines can be subdivided into an equal number, say n, of segments. The set of $4(n+1)$ nodal points $(x_{jk}, y_{jk}, -k\delta)$, where $j = 1__(n+1)$ and $k = 0__3$, define $n+1$ constant $-\lambda$ lines $\lambda = \lambda_j$ in the parametric representation (58a__c) of the upper hull surface. For a given value $\lambda_j$ of $\lambda$ corresponding to any one of these constant $-\lambda$ lines, the values $x_k(\lambda_j)$ and $y_k(\lambda_j)$ of the four functions $x_k(\lambda)$ and $y_k(\lambda)$, where $k = 0__3$, can be determined by fitting cubic polynomials through the points $(x_{jk}, -k\delta)$ and $(y_{jk}, -k\delta)$, respectively, for $k = 0__3$. Specifically, the cubic approximation to the function $x(\lambda_j, z)$ takes the form

$$x(\lambda_j,z) \simeq x_{j0} + (x_{j0} - x_{j1})(z/\delta) + (x_{j0} - 2x_{j1} + x_{j2})(z/\delta)(1 + z/\delta)/2$$
$$+ (x_{j0} - 3x_{j1} + 3x_{j2} - x_{j3})(z/\delta)(1 + z/\delta)(2 + z/\delta)/6.$$

A similar cubic approximation may be defined for the function $y(\lambda_j, z)$. The values $x_k(\lambda_j)$ of the functions $x_k(\lambda)$ in equation (58a) corresponding to the foregoing cubic approximation to the function $x(\lambda_j, z)$ then are given by

$$x_0(\lambda_j) = x_{j0}, \tag{75a}$$

$$x_1(\lambda_j) = (11x_{j0} - 18x_{j1} + 9x_{j2} - 2x_{j3})/6\delta, \tag{75b}$$

$$x_2(\lambda_j) = (2x_{j0} - 5x_{j1} + 4x_{j2} - x_{j3})/2\delta^2, \tag{75c}$$

$$x_3(\lambda_j) = (x_{j0} - 3x_{j1} + 3x_{j2} - x_{j3})/6\delta^3. \tag{75d}$$

The values $y_k(\lambda_j)$ of the functions $y_k(\lambda)$ in equation (58b) are defined in terms of the y-coordinates $y_{jk}$ of the nodal points by similar expressions.

The values $\phi_k(\lambda_j)$ of the functions $\phi_k(\lambda)$, where $k = 0__2$, in equation (59) can be determined in a similar manner by fitting a quadratic polynomial through the points $(\phi_{jk}, -k\delta)$, where $k = 0__2$ and $\phi_{jk}$ represents the value of the potential $\phi$ at the nodal point $(x_{jk}, y_{jk}, -k\delta)$. This quadratic approximation

is given by

$$\phi(\lambda_j,z) \simeq \phi_{j0} + (\phi_{j0}-\phi_{j1})(z/\delta) + (\phi_{j0}-2\phi_{j1}+\phi_{j2})(z/\delta)(1+z/\delta)/2.$$

The corresponding values $\phi_k(\lambda_j)$ of the functions $\phi_k(\lambda)$ then are given by

$$\phi_0(\lambda_j) = \phi_{j0}, \tag{76a}$$

$$\phi_1(\lambda_j) = (3\phi_{j0}-4\phi_{j1}+\phi_{j2})/2\delta, \tag{76b}$$

$$\phi_2(\lambda_j) = (\phi_{j0}-2\phi_{j1}+\phi_{j2})/2\delta^2, \tag{76c}$$

The values $\phi_k(\lambda_j)$, where k = 0__2, and the values $x_k(\lambda_j)$ and $y_k(\lambda_j)$, where k = 0__3, determined in the manner explained above for a set of n + 1 values $\lambda_j$ of the parameter $\lambda$ can be used as base-values for determining the functions $\phi_k(\lambda)$, $x_k(\lambda)$ and $y_k(\lambda)$ for a denser set of values of $\lambda$, for instance by using cubic-spline interpolation. The integral (60) defining the far-field wave-amplitude function can then be evaluated numerically by dividing the top waterline into a sufficiently-large number, say N, of segments which may be treated as straight and within which the amplitude function $a_\pm$ may be regarded as constant. Specifically, the integral (60) becomes

$$K(t) \simeq q^2 \sum_{j=1}^{N} (\lambda_{j+1}-\lambda_j)\,(a_j{}^{+}I_j{}^{+} + a_j{}^{-}I_j{}^{-}), \tag{77}$$

where $a_j{}^{\pm}$ represents the mean value of the amplitude-function $a_\pm$ within the segment $[\lambda_j,\lambda_{j+1}]$, and $I_j{}^{\pm}$ is the integral defined as

$$I_j{}^{\pm} = E_j{}^{\pm} \int_0^1 \exp[-i\nu^2 p(\theta_{j+1}{}^{\pm} - \theta_j{}^{\pm})\mu]\, d\mu,$$

with $E_j{}^{\pm}$ and $\theta_j{}^{\pm}$ defined as

$$E_j{}^{\pm} = \exp(-i\nu^2 p\theta_j{}^{\pm}) \text{ and } \theta_j{}^{\pm} = x_0(\lambda_j) \pm t y_0(\lambda_j). \tag{78a,b}$$

Likewise, we have

$$E_{j+1}{}^{\pm} = \exp(-i\nu^2 p\theta_{j+1}{}^{\pm}) \text{ and } \theta_{j+1}{}^{\pm} = x_0(\lambda_{j+1}) \pm t y_0(\lambda_{j+1}). \tag{79a,b}$$

The integral $I_j{}^{\pm}$ may readily be evaluated analytically, with the result

$$I_j{}^{\pm} = (E_j{}^{\pm} + E_{j+1}{}^{\pm})/2 \quad \text{if } \theta_j{}^{\pm} = \theta_{j+1}{}^{\pm}, \tag{80a}$$

$$I_j{}^{\pm} = iF^2 q(E_{j+1}{}^{\pm} - E_j{}^{\pm})/(\theta_{j+1}{}^{\pm} - \theta_j{}^{\pm}) \quad \text{if } \theta_j{}^{\pm} \neq \theta_{j+1}{}^{\pm}. \tag{80b}$$

The number of segments into which the top waterline must be divided increases as the Froude number decreases [2]. The foregoing numerical method thus becomes less efficient as F becomes smaller. However, it obviously is considerably more efficient to evaluate the (one-fold) waterline integral (60) than the (two-fold) surface integral involved in the basic expression (6) for the far-field wave-amplitude function. Furthermore, an analytical approximation to the integral (60) may be obtained by taking advantage of the rapid oscillations of the exponential function $E_0^{\pm}$ at small values of the Froude number, as was already noted. This complementary asymptotic approximation is presented in part 2 of this study.

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| 3 | 1543 | |
| 3 | 1544 | |
| 2 | 156 | |
| 1 | 1561 | |
| 1 | 1562 | |
| 1 | 1563 | |
| 1 | 1564 | |
| 1 | 16 | |
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| 10 | 5211.1 | Reports Distribution |
| 1 | 522.1 | TIC (C) |
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